N89-25230

Structural Damage Assessment as an Identification Problem *

P. Hajela and F.J. Soeiro
Aerospace Engineering, Mechanics & Engineering Science
University of Florida, Gainesville, Florida

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* Paper was submitted for presentation at conference but could not be presented due to time restrictions.

Abstract

Damage assessment of structural assemblies is treated as an identification problem. A brief review of identification methods is first presented with particular focus on the output error approach. The use of numerical optimization methods in identifying the location and extent of damage in structures is studied. The influence of damage on eigenmode shapes and static displacements is explored as a means of formulating a measure of damage in the structure. Preliminary results obtained in this study are presented and special attention is directed at the shortcomings associated with the nonlinear programming approach to solving the optimization problem.

Introduction

Structural systems in a variety of applications including aerospace vehicles, automobiles, civil engineering structures such as tall buildings, bridges and offshore platforms, accumulate damage during their service life. From the standpoint of both safety and performance, it is desirable to monitor the occurrence, location, and extent of such damage. System identification methods, which may be classified in a general category of nondestructive evaluation techniques, can be employed for this purpose. Using experimental data, such as eigenmodes, static displacements and damping factors, and an analytical structural model, parameters of the structure such as its mass, stiffness and damping characteristics can be identified. The approach is one where the structural properties of the analytical model are varied to minimize the difference between the analytically predicted and empirically measured response. In this process, the number of equations describing the system is typically different from the number of unknowns, and the problem reduces to obtaining the best solution from the available data.

The genesis of identification methods in structural analysis can be traced to the model determination and model correction problems. The underlying philosophy in these efforts was that a reasonably close analytical model of the structural system was available, and that deviations in the analytical response from the measured response could be used to implement corrections in the model to account for these variations. This resulted in the adoption of a standard strategy wherein the change in the analytical model was minimized to obtain a match between analytical and empirical data. For the class of problems for which it was intended, the method has enjoyed a fair degree of success. A large body of identification work has been based on measurements of eigenmodes of the structure, and this is philosophically the right approach as eigenmodes reflect the global behavior of the structure. However, some reservations about this approach do exist, as these modes are sensitive to the physical boundary conditions that exist on the experimental model, and are not accounted for in the analytical model.

The problem of damage assessment in structures by identification methods is similar to the one described above. However, the approach of minimizing changes in the analytical model is no longer applicable, as significant variations can be introduced locally due to damage in a structural component. In the present work, damage in the structure is represented by a reduction in the elastic properties of the material, and these are designated as design variables of the optimization problem. Both eigenmodes and static structural deflections are used in the identification process. The use of static structural displacements as the measured response is a departure from the standard practice of using vibration modes. Since these displacements are reflective of the applied loading, an auxiliary problem in their use is one of determining the load conditions which are best suited for a global model identification.

Subsequent sections of this paper are devoted to the review of identification methods and their prior applications in structural damage assessment. The approach used in the current work is then presented, with special emphasis on the numerical efficiency aspects of the optimization problem. In particular, the use of a reduced set of dominant design variables and constructing equivalent reduced order models for damage assessment is explored with some success.

System Identification

The veracity of analytical models is usually determined by comparing the response predicted by the model with the response observed in tests or during operation. Although measurements are in themselves imprecise due to the equipment errors and data acquisition techniques used, reasonable bounds can be imposed within which the experimental data is expected to lie. The difference between the measured and analytical data may be large enough to be considered unacceptable. In this case, if there is sufficient confidence in the experimental data, identification methods can be invoked to improve the analytical model. This subject is not new and several studies pertinent to the field are documented in the literature [1-5]. In some cases, experimental data may even be used to deduce an analytical model which eludes analytical derivation.

The data utilized in identification may include both input and output measurements, or, some system dependent characteristics such as modal parameters, which in turn are functions of the input-output measurements. A priori knowledge about the behavior of the system may also be available in the form of an analytical model. In the case of discrete structural dynamic systems, the model consists of linear second order differential equations. The mass, damping and stiffness matrix elements constitute the parameters to be identified.

Identification techniques may be classified in many different ways. Such classification is typically based on the type of data used, on the type of system being identified, or on the type of formulation employed [6]. Three of the more important formulations used in identification of structural systems are briefly discussed in the following paragraphs. These are the equation error approach, the output error approach and the minimum deviation approach.

In the equation error approach, equations describing the system response are explicitly stated. The system parameters, which are typically coefficients in such equations, are then selected to minimize the error in satisfying the system equations with a set of measured input-output data. Consider a linear differential equation represented in a functional form as follows.

$$f(c_1,c_2,x,t)=g(t) \tag{1}$$

Here, c_1 and c_2 are considered as the unknown system parameters, and $x(t)$ and its derivatives represent the system response at a time t; $g(t)$ is the forcing function. The system response and the loading is explicitly measured over some characteristic time period. An objective function, which is the measure of residual errors in the system equations for given values of the parameters, is formulated as follows:

$$F = \{ \int_0^T (f(c_1,c_2,x_m,t) - g_m(t))^2 \, dt \} \tag{2}$$

This function is then extremized by differentiating with respect to each system parameter and equating to zero. The approach results in the same number of equations as coefficients to be determined and is therefore regarded as a direct method. In eqn. (2), subscript m denotes measured response, and T denotes the characteristic period over which measurements are made.

The output error approach selects some system characteristic response as the entity for which a match between the analytical prediction and experimental measurements is considered to reflect a good analytical model. An objective function is formulated that is typically an averaged least-squares measure as follows.

$$F = \int_0^T (x_m(t) - x(t))^2 \, dt \tag{3}$$

The analytical model from which x(t) is obtained, contains system parameters which are adjusted to minimize the function F. In structural dynamics identification problems, system eigenmodes are generally selected as the characteristic response quantities used to identify the model.

The minimum deviation approach is frequently used in structural identification problems. In this approach, deviation of the system parameters from initial assumed values is minimized, subject to the constraints that the system equations be satisfied. An illustration of this approach in structural applications is in the determination of changes in the elastic stiffness matrix. In such applications, the mass matrix is assumed to be accurately defined. A weighted matrix norm of the difference between the a priori and corrected stiffness matrices is minimized in the identification process. This norm can be written as

$$F = \| M^{-1/2} (K - \bar{K}) M^{-1/2} \| \tag{4}$$

where M is the mass matrix, K is the desired stiffness matrix, and $\bar{K}$ is the a priori stiffness matrix. This minimization is subject to the constraint that the modified stiffness matrix remain symmetric. An incomplete set of eigenmodes is measured, and these eigenmodes are required to satisfy the eigenvalue equation and be orthogonal to the modified stiffness matrix. This results in the following equality constraints:

$$\phi^T K \phi = \Omega^2 \tag{5}$$

$$K = K^T \tag{6}$$

$$K\phi = M\phi\Omega^2 \tag{7}$$

In the above equations, ϕ is an nxp modal matrix and Ω^2 is a diagonal matrix of eigenvalues for the p measured modes; n is the number of degrees of freedom for the structural dynamic system. The constraints are incorporated into the objective function by means of Lagrange multipliers, and the application of the optimality condition yields a close form expression for the corrected stiffness matrix as follows:

$$K = K - K\phi\phi^T M - M\phi\phi^T K + M\phi\phi^T K\phi\phi^T M + M\phi\Omega^2\phi^T M \tag{8}$$

Damage Assessment

The foregoing discussion outlines various identification techniques and their applicability in predicting changes in a structural configuration. This approach has been employed for detecting changes in the analytical model due to structural damage. The requirements of the identification problem are to use experimental data to determine if the structure is damaged and to further detect the extent and location of that damage.

A major structural failure in the form of a macroscopic rupture can be visually observed. However, changes in the structural load carrying capacity that is localized to an internal structural component may not be detected as easily. As weight considerations dictate the use of lighter, more flexible and actively stiffened structures, it has become increasingly important to develop a consistent approach that would allow real time detection and correction for structural damage.

The use of identification techniques to detect damage has been recently attempted [7-8], but with limited success. Chen and Garba [7] discuss the use of measured eigenmodes to determine the changes in the stiffness matrix, assuming no changes in the mass matrix with damage. They employ the use of a direct optimization method, in which an Euclidean norm of the changes in the stiffness matrix is minimized, subject to the constraints that the modified system matrices produce the measured eigenmodes. This is an application of the minimum deviation approach described in the previous section. The number of unknown elements of the modified stiffness matrix is typically much larger than the number of equations available. An infinite number of solutions is possible in the optimization problem, and this difficulty is clearly evidenced by the results obtained.

Smith and Hendricks [8] report the evaluation of a similar method and another that uses linear perturbations of system submatrices and an energy distribution analysis to detect damage. Both methods show appreciable problems in detecting damage. Another shortcoming of these methods, based on the minimum deviation approach, is the fact that it fails in showing damage clearly. The stiffness matrix has several entries which depend on the elastic and geometric properties of the structure as well as on structural element connectivity. There are overlaps in the matrix due to the contribution of different members sharing the same node, which makes it difficult to identify where damage is occurring. Also, the minimum deviation approach tries to deviate the minimum from the a priori model. In this kind of problem, the a priori model is the original stiffness matrix corresponding to the undamaged structure. The damage may be quite severe and located in different members of the structure. The changes in the stiffness matrix may be significant, thereby increasing the possibility that this approach may not be able to give good results.

In a finite element formulation, structural characteristics are defined in terms of the stiffness, damping, and mass matrices [K], [C] and [M], respectively. The governing equation of equilibrium for a dynamical system involves each of these matrices, and can be written as

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = \{P(t)\} \tag{9}$$

where $\{x\}$ is the displacement vector, and $\{P(t)\}$ is the vector of applied loads. The eigenvalue problem can be stated in terms of the system matrices, eigenvalues ω_i^2, and the corresponding eigenmodes ϕ_i as follows:

$$([K] - \omega_i^2 [M]) \{\phi_i\} = \{0\} \tag{10}$$

The static load-deflection relation only involves the system stiffness matrix.

$$[K]\{x\} = \{P\} \tag{11}$$

It is clear from these equations that a change in the system matrices results in a changed response, and this difference can be related to changes in specific elements of the system matrices. Since internal structural damage typically does not result in a loss of material, we will assume the mass matrix to be a constant. The stiffness matrix can be expressed as a function of the sectional properties A,I, and J, the element dimensions denoted by t and L, and by extensional and shear modulii E and G, respectively.

$$[K] = [K(A,I,J,L,t,E,G)] \tag{12}$$

In the present work, changes in these quantities due to damage are lumped into a coefficient d_i that is used to multiply the extensional modulus E_i for the particular element. These d_i's constitute the design variables for the optimization problem. If the measured and analytically determined static displacements or eigenmodes are denoted by $\{y_m\}$ and $\{y_a\}$, respectively, the optimization problem can be stated as finding a vector of d_i (and hence the analytical stiffness matrix) that minimizes the quantity

$$\sum_i \sum_j (y_m^{ij} - y_a^{ij})^2 \tag{13}$$

where i represents the degree of freedom, and j denotes the static loading condition or a particular eigenmode. This minimization requires that $\{y_a\}$ be obtained from the eigenvalue problem or the load deflection equations using the [K] matrix that must be identified. Lower and upper bounds of 0 and 1 were established for the design variables d_i.

The nonlinear programming solution to the damage detection problem can be computationally demanding, and approximations were used to circumvent this problem. The first approach was one in which a select number of dominant variables were used in the optimization, based on the magnitude of the search direction. This set of dominant variables was periodically revised with a new assessment of the dominance.

The second approach was one in which equivalent reduced order models were constructed. Consider the truss shown in Figure 1, subjected to tensile and bending loads. An equivalent beam model (figure 2), with an independent axial and bending stiffness for each section, can be obtained to simulate the behavior of the truss structure. Each section corresponds to one bay in the truss. The degrees of freedom and number of design variables for the beam are 15 and 5, and this compares with 20 degrees of freedom and 25 design variables for the original truss structure. The equivalent section is first used to identify the section in which damage exists. This gives us a reduced set of variables to work with in the actual structure, and convergence to the correct stiffness matrix is far more efficient.

Recognition of the fact that measured data often cannot be obtained for all nodes and degrees of freedom, the identification problem was also carried out with a reduced set of measurements. The results of this implementation were encouraging.

In working with static displacements as the measured response, careful consideration must be given to the fact that the applied loading is not one that allows only a few members of the structural system to participate in the load carrying process. This issue has been studied in extensive detail and results are presented in [9].

Numerical Examples

The procedure described in the preceding section was implemented on a VAX 11-750 computer. The Davidon-Fletcher-Powell variable metric method was used for function minimization and a finite element analysis program EAL [10] was used to obtain the structural response. The simulated measured data was the finite element solution obtained for the damaged structure, corrupted by a random noise signal.

The method developed for damage detection has been applied to a series of representative truss models and semimonocoque structures. A twenty five bar truss shown in Fig.1 was damaged in element 11 by reducing its Young's modulus to 0.0 . The first four eigenmodes were used to detect the extent and location of damage in the structure, and the results are shown in Table 1. The same problem was solved using static displacements for the indicated static loading, and these results are shown in Table 2. This example was repeated with the use of master design variables and was more efficient from the standpoint of computational effort. An equivalent beam with five elements, each corresponding to one bay of the truss is created (Fig.2). The problem is then solved in two steps. The first step is the use of the reduced order model to detect the area of damage in the five element beam. The second step entails the detection of damage in the original structure with reduced number of design variables.

Another representative example is that of a semimonocoque wing box structure (Fig.3) consisting of axial rod elements and membrane elements. Membrane element 2 was damaged and the identification of damage conducted with the use of static displacements. Table 3 summarizes the results for this example, clearly indicating the need for applying a torque that forces the membrane to participate more equitably in the load bearing process. This case also represents the successful use of a reduced set of experimental measurements in the damage detection process. In this 54 d.o.f system, only 9 displacements were employed: 3 horizontal displacements in the middle of the upper panel and 3 vertical displacements at each edge of the upper panel. In using static displacements for damage assessment purposes, critical members are more easily detected. This is reassuring from the standpoint of safety in the structure.

The iterative optimization methods used in this approach are clearly susceptible to convergence to a local optimum. One approach that circumvents the problem of nonconvexities in the design space [11] is presently under development and will be used in future work.

Conclusions

This paper presents an approach for damage detection in structures based on identification techniques. The different identification methods are discussed with particular focus on the output error approach. Both eigenmodes and static displacements are used in the identification procedure, with the static displacements providing the advantage of lower computational cost and easier measurement. Approximation concepts have been introduced to decrease the computational cost such as equivalent structures with less d.o.f and master displacements. The approach has given extremely encouraging results and has proved to be very flexible. Future work will include the study of damage detection in composite materials that are extensively used in aerospace structures. Damping also promises to be a good parameter for damage assessment as it represents an energy dissipation process that may be influenced by microscopic or macroscopic damage.

References

1. A. Berman and W.G. Flannelly, "Theory of Incomplete Models of Dynamic Structures", AIAA Journal, Vol. 9 ,No. *, Aug 1971, pp1481-1487.

2. A. Berman and E.J. Nagy, "Improvement of a Large Analytical Model Using Test Data", AIAA Journal, Vol. 21, No. 8, 1983, pp1168-1173.

3. M. Baruch, "Optimal Correction of Mass and Stiffness Matrices Using Measured Modes", AIAA Journal, Vol. 20, No.11, 1982, pp1623-1626.

4. A.M. Kabe, "Stiffness Matrix Adjustment Using Mode Data", AIAA Journal, Vol.23, No. 9, 1985, pp1431-1436.

5. C.B. Yun and M. Shinozuka, "Identification of Nonlinear Structural Dynamic Systems", Journal of Structural Mechanics, Vol. 8, 1980, pp187-203.

6. S. Hanagud, M.Meyyappa and J.I. Craig, "Identification of Structural Dynamic Systems", Recent Trends in Aeroelasticity, Structures and Structural Dynamics, Florida Presses, 1987.

7. J.C. Chen and J.A. Garba, "On Orbit Damage Assessment for Large Space Structures", Proceedings of the 28th SDM Conference, Monterey, California, April 1987.

8. S.W. Smith and S.L. Hendricks, "Evaluation of Two Identification Methods for Damage Detection in Large Space Structures", Proceedings of the 6th VPI&SU/AIAA Symposium on Dynamics and Control of Large Structures, 1987.

9. P. Hajela and F.J. Soeiro, "Structural Damage Detection Based on Static and Modal Analysis", in review for the 30th AIAA/ASME/ASCE/AHS/ASC SDM Conference, Mobile, Alabama, April 1989.

10. D. Whetstone, "SPAR - Reference Manual", NASA CR-145098-1, February 1977.

11. P. Hajela, "Genetic Search - An Approach to the Nonconvex Optimization Problem", in review for presentation at the 30th AIAA/ASME/AHS/ASCE SDM Conference, Mobile, Alabama, April 1989.

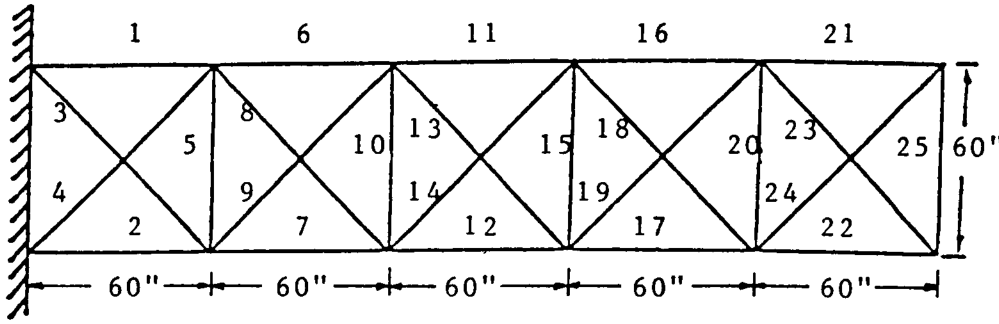

Fig.1 Twenty five bar planar truss

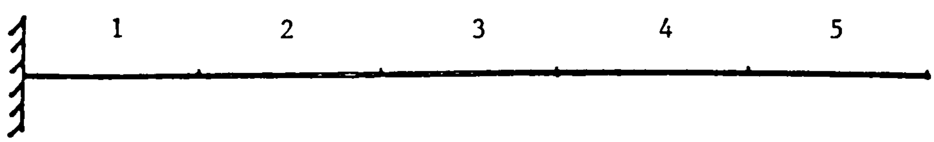

Fig.2 Equivalent five element beam model for the twenty five bar truss

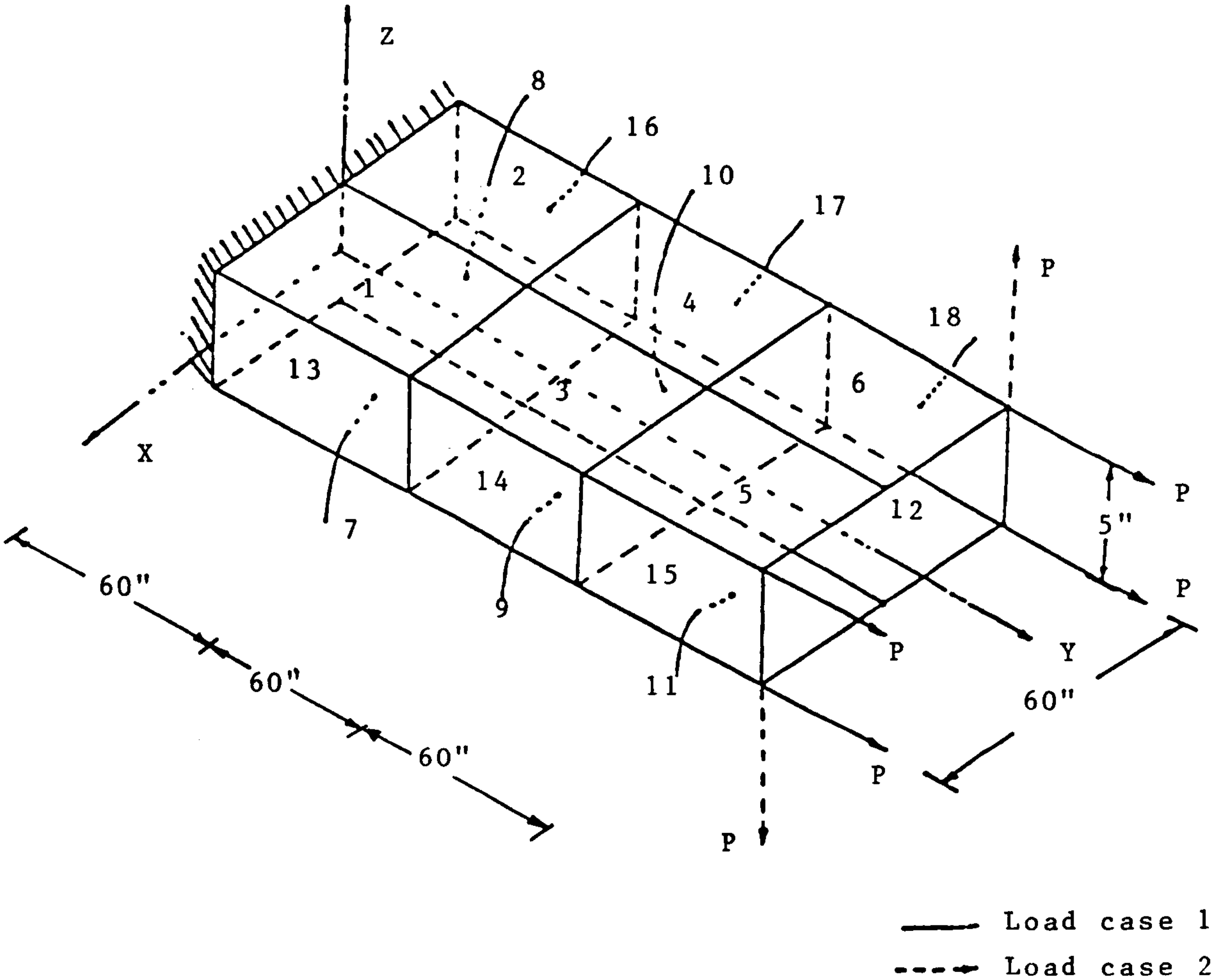

Fig.3 Semimonocoque wing box model

OPTIMIZATION RESULTS

VARIABLE	LOWER BOUND	VALUE	UPPER BOUND
1	0.00000E+00	0.92536E+00	0.10000E+01
2	0.00000E+00	0.93169E+00	0.10000E+01
3	0.00000E+00	0.93320E+00	0.10000E+01
4	0.00000E+00	0.91196E+00	0.10000E+01
5	0.00000E+00	0.96094E+00	0.10000E+01
6	0.00000E+00	0.87899E+00	0.10000E+01
7	0.00000E+00	0.96327E+00	0.10000E+01
8	0.00000E+00	0.10000E+00	0.10000E+01
9	0.00000E+00	0.87905E+00	0.10000E+01
10	0.00000E+00	0.98613E+00	0.10000E+01
11	0.00000E+00	0.00000E+00	0.10000E+01
12	0.00000E+00	0.89017E+00	0.10000E+01
13	0.00000E+00	0.85975E+00	0.10000E+01
14	0.00000E+00	0.91797E+00	0.10000E+01
15	0.00000E+00	0.10000E+01	0.10000E+01
16	0.00000E+00	0.85562E+00	0.10000E+01
17	0.00000E+00	0.90889E+00	0.10000E+01
18	0.00000E+00	0.92100E+00	0.10000E+01
19	0.00000E+00	0.98278E+00	0.10000E+01
20	0.00000E+00	0.10000E+01	0.10000E+01
21	0.00000E+00	0.10000E+01	0.10000E+01
22	0.00000E+00	0.93679E+01	0.10000E+01
23	0.00000E+00	0.93863E+01	0.10000E+01
24	0.00000E+00	0.94524E+01	0.10000E+01
25	0.00000E+00	0.10000E+01	0.10000E+01

Table 1. Twenty five bar truss - results using eigenmodes

OPTIMIZATION RESULTS

VARIABLE	LOWER BOUND	VALUE	UPPER BOUND
1	0.00000E+00	0.10000E+01	0.10000E+01
2	0.00000E+00	0.10000E+01	0.10000E+01
3	0.00000E+00	0.10000E+01	0.10000E+01
4	0.00000E+00	0.10000E+01	0.10000E+01
5	0.00000E+00	0.99410E+00	0.10000E+01
6	0.00000E+00	0.74186E+00	0.10000E+01
7	0.00000E+00	0.10000E+00	0.10000E+01
8	0.00000E+00	0.10000E+00	0.10000E+01
9	0.00000E+00	0.10000E+00	0.10000E+01
10	0.00000E+00	0.98792E+00	0.10000E+01
11	0.00000E+00	0.17763E-01	0.10000E+01
12	0.00000E+00	0.91020E+00	0.10000E+01
13	0.00000E+00	0.99999E+00	0.10000E+01
14	0.00000E+00	0.94951E+00	0.10000E+01
15	0.00000E+00	0.99196E+00	0.10000E+01
16	0.00000E+00	0.73150E+00	0.10000E+01
17	0.00000E+00	0.90617E+00	0.10000E+01
18	0.00000E+00	0.95281E+00	0.10000E+01
19	0.00000E+00	0.98696E+00	0.10000E+01
20	0.00000E+00	0.10000E+01	0.10000E+01
21	0.00000E+00	0.96812E+01	0.10000E+01
22	0.00000E+00	0.99026E+00	0.10000E+01
23	0.00000E+00	0.96977E+00	0.10000E+01
24	0.00000E+00	0.95638E+00	0.10000E+01
25	0.00000E+00	0.99999E+00	0.10000E+01

Table 2. Twenty five bar truss - results using static displacements

Element No. (Panel)	Design Variables (d_i) using static response		
	Load 1 only	Load 1 & 2	Exact sol.
1	1.0000	1.0000	1.0000
2	0.1240	0.1020	0.1000
3	0.9601	1.0000	1.0000
4	0.7720	1.0000	1.0000
5	0.9585	1.0000	1.0000
6	0.9837	0.9999	1.0000
7	0.9736	1.0000	1.0000
8	1.0000	1.0000	1.0000
9	0.9954	1.0000	1.0000
10	0.8713	1.0000	1.0000
11	0.9979	1.0000	1.0000
12	0.8984	1.0000	1.0000
13	0.9999	1.0000	1.0000
14	0.9998	1.0000	1.0000
15	0.9981	1.0000	1.0000
16	0.9926	0.9702	1.0000
17	1.0000	1.0000	1.0000
18	0.9855	0.9999	1.0000

Table 3. Results for the 54 d.o.f. semimonocoque wing box model using static displacements for damage detection